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By solvng the transport equations for the amplitude of the progressing wave expansion of Maxwell's equations using the Lorentz model, ~~we find that~~ the amplitudes do <u>not</u> decay exponentially along the ray. Thus, ~~we are~~ now studyings more general models of the dispersive media, to determine the qualitative features for which the amplitudes do, or do not decay exponentially. Preliminary results suggest that a classification of dispersive media is obtained depending on the relative orders of the differential operators.

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For a dispersive, non-magnetic, electrically conducting medium, Maxwell's equations for the propagation of electromagnetic waves are given in terms of dimensionless variables by

$$\nabla \times \underset{\sim}{E} = - \underset{\sim}{H}_t \quad , \quad \nabla \times \underset{\sim}{H} = \underset{\sim}{D}_t \quad , \tag{1}$$

$$\underset{\sim}{D} = (\underset{\sim}{E} + \underset{\sim}{P}) \quad . \tag{2}$$

The vectors $\underset{\sim}{D}(\underset{\sim}{x},t)$, $\underset{\sim}{E}(\underset{\sim}{x},t)$, $\underset{\sim}{H}(\underset{\sim}{x},t)$ and $\underset{\sim}{P}(\underset{\sim}{x},t)$ are proportional to the electric displacement, the electric field strength, the magnetic field strength, and the polarization vector, respectively. The constitutive properties of the medium, which are experimentally determined give the polarization vector in (2) as a linear functional of $\underset{\sim}{E}$. A general expression is given by the differential constitutive law,

$$\underset{\sim}{P}(\underset{\sim}{x},t) = \sum_{j=1}^{N} \underset{\sim}{P}_j(\underset{\sim}{x},t) \quad , \tag{3a}$$

$$\sum_{n=0}^{r} \alpha_{nj} \frac{\partial^n \underset{\sim}{P}}{\partial t^n} = \sum_{m=0}^{s} \beta_{mj} \frac{\partial^m \underset{\sim}{E}}{\partial t^m} \quad , \quad j = 1,2,\ldots N \quad . \tag{3b}$$

The specified constitutive coefficients, α_{nj} and β_{mj}, $m = 0,1,\ldots s$, $n = 0,1,\ldots,r$ and $j = 1,2,\ldots,N$ are experimentally determined.

More generally, we express $\underset{\sim}{P}$ as a linear functional of $\underset{\sim}{E}$ by the heredity integral,

$$\underset{\sim}{P}(\underset{\sim}{x},t) = \mathcal{L}\, \underset{\sim}{E}(\underset{\sim}{x},t) \equiv \int_{-\infty}^{t} \underset{\sim}{E}(x,t')\epsilon(t-t')dt' \quad , \tag{3}$$

where the "memory" function, $\epsilon(\tau)$, which is experimentally determined, vanishes for $\tau < 0$.

We have first analyzed the propagation of finite jump discontinuous solutions of the system (1)-(3) using both the method of weak solutions [1,2] and progressing wave expansions [1,3] for the Debye [4] model of the

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dispersive medium. This model is believed to be a good approximation of the dispersive properties of biological materials. Thus, each $\underset{\sim}{P}_j$ satisfies a first order ordinary differential equation

$$\frac{\partial}{\partial t}\underset{\sim}{P}_j + \alpha_j \underset{\sim}{P}_j = \beta_j \underset{\sim}{E} \quad , \quad j = 1,2,\ldots,N \quad . \tag{4}$$

The "constitutive" coefficients $\alpha_1,\alpha_2,\ldots,\alpha_n$ and $\beta_1,\beta_2,\ldots,\beta_n$ are to be determined experimentally, or from quantum mechanical calculations. Then,

$$\underset{\sim}{P}_j = \int_{-\infty}^{t} \underset{\sim}{E}(\underset{\sim}{x},t')\beta_j e^{-\alpha_j(t-t')} dt' \tag{5}$$

and $\underset{\sim}{P} = \int_{-\infty}^{t} \underset{\sim}{E}(\underset{\sim}{x},t) \sum_{j=1}^{N} \beta_j e^{-\alpha_j(t-t')} dt'$, so that the memory function for the Debye model is given by the linear combination of exponentially decaying functions, $\epsilon(\tau) = \sum_{j=1}^{N} \beta_j e^{-\alpha_j \tau}$.

The progressing wave expansion for the solution of (1),(2) and (5) is a representation in the form

$$\begin{aligned} \underset{\sim}{E} &= \sum_{m=0}^{\infty} f_m[\phi(\underset{\sim}{x},t)]\underset{\sim}{E}^m(\underset{\sim}{x},t) \quad , \quad \underset{\sim}{H} = \sum_{m=0}^{\infty} f_m[\phi(\underset{\sim}{x},t)]\underset{\sim}{H}^m(\underset{\sim}{x},t) \quad , \\ \underset{\sim}{P}_j &= \sum_{m=0}^{\infty} f_m[\phi(\underset{\sim}{x},t)]\underset{\sim}{P}_j^m(\underset{\sim}{x},t) \quad , \end{aligned} \tag{6}$$

where $f_0(\phi)$ is an arbitrary function of the single variable $\phi(\underset{\sim}{x},t)$ and $f_1,f_2,\ldots,$ are required to satisfy,

$$f'_m = f_{m-1} \quad , \quad m = 1,2,\ldots \quad . \tag{7}$$

Then we insert (6) into (1), (2) and (4) and use (7) for the derivatives of f_m. By equating to zero the coefficient of each function $f_m(\phi)$, we obtain a sequence of equations to determine ϕ, $\underset{\sim}{E}^m$, $\underset{\sim}{H}^m$ and $\underset{\sim}{P}_j^m$. For example, if we set

$\phi(\underset{\sim}{x},t) = t - \psi(\underset{\sim}{x})$, then ψ satisfies the eikonal equation of geometrical optics,

$$|\nabla\psi|^2 = 1 \quad , \tag{8}$$

and the coefficients $\underset{\sim}{E}^0$, $\underset{\sim}{E}^1$, etc., satisfy transport equations, which reduce to first order partial differential equations along the characteristics of (8). Solving the transport equations explicitly, shows that $\underset{\sim}{E}^0$ and $\underset{\sim}{H}^0$ decay exponentially along the rays.

The choice of the waveform $f_0(\phi)$ is determined by the type of discontinuity to be analyzed, and the form of the pulse. For example, to recover the results of our previous analysis of the propagation of jump discontinuities, we select f_0 to be the Heaviside step function i.e.

$f_0 = H(t-\psi)$.

For the Lorentz model [4] of dispersion the polarization vector is given by (3a) where the $\underset{\sim}{P}_j$ now satisfy

$$\frac{\partial^2 \underset{\sim}{P}_j}{\partial t^2} + \alpha_j \frac{\partial \underset{\sim}{P}_j}{\partial t} + \gamma_j \underset{\sim}{P}_j = \beta \underset{\sim}{E} \quad , \tag{11}$$

and the constitutive coefficients α_j, γ_j and β are to be experimentally determined. By solving the transport equations for the amplitude of the progressing wave expansion of Maxwell's equations using the Lorentz model, we find that the amplitudes do <u>not</u> decay exponentially along the ray. Thus, we are now studying more general models of the dispersive media, e.g. (3), to determine the qualitative features for which the amplitudes do, or do not decay exponentially. Preliminary results suggest that a classification of dispersive media is obtained depending on the relative orders of the differential operators on the right and left sides of (3.3b).

Finally, using the progressing wave expansion, we have determined the scattering of pulses from dispersive half spaces and from dispersive targets.

A paper entitled "Progressing Wave Expansions for Temporally Dispersive Electromagnetic Waves and the Classification of Dispersive Media" is currently in preparation.

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